Electromagnetic Foundation of the Mo-Papas Theory

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Dirac's method for deriving an equation of motion of the radiating electron is used to deduce from Maxwell's theory the Mo-Papas equation in place of the Lorentz-Dirac equation. The emergence of the Schott term is avoided by means of a suitable assumption about the extension of the electron in space-time. Since one deals in this approach with a finite-size particle, no infinite terms arise in connection with the particle's mass. Moreover, asymptotic conditions are not necessary, neither in the derivation of the equation of motion nor in determining its solutions.

I. Introduction and Survey of Results

In a preceding part [1] of the current studies of "the problem of radiation reaction in classical electrodynamics" we have investigated the following nonlocal equation of motion * for the radiating electron proposed in an earlier paper [2]

$$m c^2 [\hat{u}^\lambda - (u \, \hat{u}) u^\lambda] = K^\lambda \qquad (\mathrm{I}, 1)$$

where $\hat{u}^\lambda{}_{(s)} \equiv \dot{u}^\lambda{}_{(s - \Delta s)}$. It has been shown in these papers that this equation excludes runaway solutions and pre-acceleration effects, and it is hoped therefore that this equation of motion will provide a quite reasonable and satisfactory theory of the classical radiating electron.

In the preceding part [1] we have also demonstrated the various relations between the new equation (I, 1) and other equations of motion found in the literature, e. g. the finite-differences theory of Caldirola [3], Mo-Papas theory [4], and that of Lorentz-Dirac-Rohrlich [5, 6]. Though the basic equation

$$m c^2 \dot{u}^\lambda = K^\lambda + \tfrac{2}{3} Z^2 [\ddot{u}^\lambda + (\dot{u} \, \dot{u}) u^\lambda] \qquad (\mathrm{I}, 2)$$

(Lorentz-Dirac equation)

of the latter theory could be derived from the non-local Eq. (I, 1) under certain approximation assumptions, this derivation did not appear as an especially reliable one. The reason for this failure of Eq. (I, 2) as an approximative form of (I, 1) is, that the Lorentz-Dirac equation admits solutions (runaway; preaccelerative part of an otherwise reasonable solution), for which the neglected higher derivative terms are not small compared to those

* The notation is here the usual one, as in all preceding papers, i. e. world line is given by $x^\lambda = z^\lambda{}_{(s)}$, four-velocity $c \, u^\lambda = c \, (dz^\lambda/ds)$, $u^\lambda u_\lambda \equiv (u \, u) = (u^0)^2 - \mathbf{u}^2 = +1$.

terms entering the approximative equation of motion (I, 2)!

In addition to the Lorentz-Dirac equation we have deduced a further approximative form of the "exact" equation of motion (I, 1), which is free of the unphysical traits of the Lorentz-Dirac equation and which can therefore be looked upon as the correct approximation equation of (I, 1):

$$m c^2 \dot{u}^\lambda + \frac{2}{3} \frac{Z^3}{m c^2} (F^{\mu\nu} \dot{u}_\mu u_\nu) u^\lambda \qquad (\mathrm{I}, 3)$$
$$= Z F^{\mu\lambda} u_\mu + \frac{2}{3} \frac{Z^3}{m c^2} F^{\mu\lambda} \dot{u}_\mu .$$

This is the equation of Mo and Papas [4]. The procedure, resulting in this equation, consisted in a Taylor series expansion of the kinematical quantities with respect to the non-locality parameter Δs about a suitably chosen reference point.

It is essential for the following considerations, that a total neglection ($\Delta s \to 0$) of the non-locality in (I, 1) would lead to the neutral particle limit

$$m c^2 \dot{u}^\lambda = K^\lambda . \qquad (\mathrm{I}, 4)$$

Taking account for the non-local character of the equation of motion (I, 1) in lowest order of the size parameter Δs ($= \tfrac{2}{3} Z^2/m c^2$) yields at once the Mo-Papas equation (I, 3), which exhibits therefore the radiation reaction phenomenon in lowest order. Since the non-locality of the equation of motion is assumed to be an expression of the finite extension of the electron, we thus recognize clearly that *radiation reaction* and *finite size* of the electron are intimately connected with each other. From this point of view, Dirac's original intention [5], namely to find a consistent radiation reaction theory of the classical *point* electron, was doomed to failure from the very beginning. The unphysical effects were the penalty for this inconsistent attempt.

On the other hand, Dirac has made use of Maxwell's theory of electromagnetism as far as possible, and one could conclude, that under the exclusive use of Maxwell's theory the emergence of the Lorentz-Dirac equation be uniquely determined. But this conclusion is wrong. Indeed, we shall show in the present paper that it is exactly Dirac's method of deriving an equation of motion for the radiating electron, which leads to the Mo-Papas equation and not to the Lorentz-Dirac equation. The crucial point is, that we do not think of a *point* electron, but we intend to derive an equation of motion for a *very small but finite-sized* electron!

Since the Dirac method deals with certain electromagnetic fluxes through appropriately chosen surfaces in Minkowski space, we will provide first some mathematical tool in the next section.

II. Some Geometrical Preliminaries

An important step of Dirac's method consists in the calculation of the electromagnetic flux through some tube surface surrounding the electron world line. For the point particle limit this tube is contracted on to the world line, whereby certain flux integrals become infinite. Thus it is easily understandable, that a suitable choice of the tube's shape is essential for the results. Now the work of Teitelboim [7, 8], van Weert [9], Sorg [10], Tabensky [11], Villaroel [12], and Rowe [13] demonstrates very clearly that the Schott term, being responsible for the unphysical effects, arises unavoidably whenever an orthogonal hyperplane of the electron world line is involved in the definition of the above mentioned tube. So, if we do not want to run into the well-known difficulties, we have to look for a definition of the tube without use of the orthogonal hyperplanes, thereby avoiding the emergence of the Schott term.

Such a tube (t) can be defined by the constraint of constant retarded distance R:

$$(\text{t}): \quad R = \text{const} \, (= R_0) \, . \qquad (\text{II}, 1)$$

The retarded distance R of an arbitrary event $X = \{x^\lambda\}$ from the electron world line $x^\lambda = z^\lambda_{(s)}$ is itself defined by (see Fig. 1)

$$R = u_{\lambda(s)} \, (x^\lambda - z^\lambda_{(s)}) \, , \qquad (\text{II}, 2)$$

where $\{z^\lambda_{(s)}\}$ is the intersection of the backward light cone (vertex in X) with the world line. The space-like unit vector $\{v^\lambda\}$ points from this intersection $\{z^\lambda_{(s)}\}$ to the projection X' of X into the
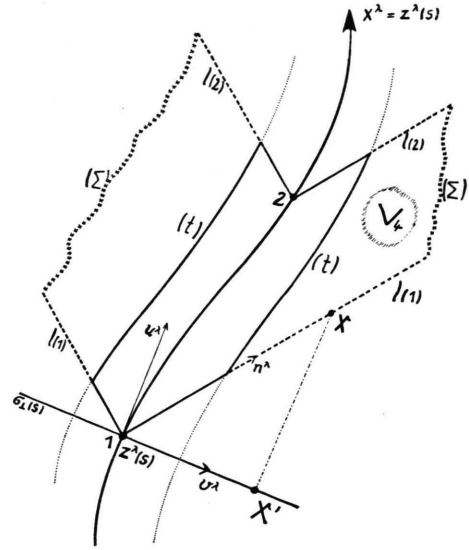

Fig. 1

orthogonal hyperplane $\sigma_{\perp(s)}$ of the world line in $\{z^\lambda_{(s)}\}$. The null vector $\{n^\lambda\}$

$$n^\lambda = (x^\lambda - z^\lambda_{(s)})/R \qquad (\text{II}, 3\text{a})$$

is now decomposable as

$$n^\lambda = u^\lambda + v^\lambda; \quad n^\lambda n_\lambda = 0 \, . \qquad (\text{II}, 3\text{b})$$

In the hyperplane $\sigma_{\perp(s)}$ one can choose an orthonormal triad of space-like vectors such that $\{v^\lambda\}$ can be described by two polar angles ($\{\Theta, \Phi\}$ with respect to this triad. Thus, an arbitrary point $\{\xi^\lambda\}$ on the tube $R = R_0$ is given by

$$\xi^\lambda_{(s, \Theta, \Phi)} = z^\lambda_{(s)} + R_0 \cdot n^\lambda_{(\Theta, \Phi)} \, . \qquad (\text{II}, 4)$$

This tube has been used by several workers with the same purpose as ours. But the assumptions and results obtained are very different, as well with respect to each other as to the present result. Either one comes to an equation of motion with ever decreasing rest mass of the electron (Marx [14]), or one rederives the Lorentz-Dirac equation by some inconsistent procedure (Hogan [15]), by modifying the usual form of the electromagnetic energy-momentum tensor (Synge [16]), and by reintroducing the Schott term on non-electromagnetic grounds (Bhabba [17]). Neither the variable rest mass nor the unphysical effects of the rederived Lorentz-Dirac equation are especially attractive.

In the next sections, we will present a new approach avoiding all these unwanted effects.

III. Calculation of Electromagnetic Fluxes

As indicated in the previous section, we are dealing in the following with a finite-size electron. Such a thing cannot be described by the linear Maxwell theory in a covariant and satisfactory way. Instead of the many attempts in literature (see Erber[18]), which refer to a rigid charged sphere, we prefer the approach of Euler[19], who has shown by explicit cross section calculations for the scattering of light by light, that the adequate classical way of description of the virtual electron-positron pair creation and annihilation processes of the scattering should be a non-linear electromagnetic theory, e. g. of the Born-Infeld type[20] (A brief appreciation of Euler's ideas was given recently by Wentzel[21]). But as long as nobody is able to show how such an improved (non-linear) theory of the classical radiating electron arises from a classical limit of quantum electrodynamics[22], it seems to us to be legitimate to deal with a linear cut-off theory instead of the (presently unknown) non-linear one. At least in lowest order of an expansion of the radiation reaction theory in demand with respect to the extension parameter, which has to enter the hypothetical non-linear theory but corresponds in the present cut-off theory to the retarded tube radius R_0 of the previous section, one can expect good agreement between that hypothetical non-linear theory and the cut-off theory preferred hereafter for the sake of simplicity.

According to this philosophy, we shall describe our classical electron by the well-known Liénard-Wiechert potentials[6] outside and on the tube (t), whereas no electromagnetic statements about the interior of the tube are necessary, because the Dirac method makes use only of the flux through the tube surface. The contribution of the interior to the energy-momentum of the electron is considered of mechanical (i. e. non-electromagnetic) nature and incorporated in such a way that the resulting equation of motion becomes meaningful.

Hence, our next task is to calculate the flux

$$[P_{\text{tot}}^\mu]_1^2 = -\frac{1}{c} \int_1^2 {}_{(t)} T_{\text{tot}}^{\mu\nu} \, d^3f_\nu \qquad \text{(III, 1)}$$

through the tube surface (t) between the end points 1 and 2 (see Fig. 1), where $\{T_{\text{tot}}^{\mu\nu}\}$ is the total electromagnetic energy-momentum tensor. According to the fact that this tensor is quadratic in the total field strengths $\{F_{\text{tot}}^{\mu\nu}\}$

$$-4\pi T_{\text{tot}}^{\mu\nu} = F_{\text{tot}}^\mu{}_\lambda F_{\text{tot}}^{\nu\lambda} - \tfrac{1}{4} g^{\mu\nu} (F_{\text{tot}}^{\varrho\sigma} F_{\text{tot}\,\varrho\sigma}) \qquad \text{(III, 2)}$$

and these in turn are a sum of the retarded Liénard-Wiechert fields $\{F_p^{\mu\nu}\}$ of the particle and of the external fields $\{F_e^{\nu\mu}\}$:

$$F_{\text{tot}}^{\mu\nu} = F_p^{\mu\nu} + F_e^{\mu\nu}, \qquad \text{(III, 3)}$$

the total energy-momentum tensor $\{T_{\text{tot}}^{\mu\nu}\}$ is a sum of three terms

$$T_{\text{tot}}^{\mu\nu} = T_p^{\mu\nu} + T_{\text{int}}^{\mu\nu} + T_e^{\mu\nu}. \qquad \text{(III, 4)}$$

The purely external contribution from $\{T_e^{\mu\nu}\}$ is neglected, because it leads to R_0^2-terms on account of the surface element[17] $\{d^3f^\lambda\}$

$$d^3f^\lambda = \{v^\lambda - R_0 (v \dot{u}) n^\lambda\} R_0^2 \, d\Omega \, ds \qquad \text{(II, 5)}$$

being at least of second power in R_0 and the external field strength $\{F_e^{\mu\nu}\}$ has no singularity on the electron's world line by assumption.

The self-field contribution in (III, 1) with (III, 4), given by

$$[P_p^\mu]_1^2 = -\frac{1}{c} \int_1^2 {}_{(t)} T_p^{\mu\nu} \, d^3f_\nu, \qquad \text{(III, 6)}$$

where $T_p^{\mu\nu}$ is constructed from the particle field $\{F_p^{\mu\nu}\}$ quite analogously to (III, 2), can be decomposed once more into a sum of the bound part

$$[P_b^\mu]_1^2 = -\frac{1}{c} \int_1^2 {}_{(t)} T_b^{\mu\nu} \, d^3f_\nu \qquad \text{(III, 7)}$$

and the emitted part

$$[P_r^\mu]_1^2 = -\frac{1}{c} \int_1^2 {}_{(t)} T_r^{\mu\nu} \, d^3f_\nu \qquad \text{(III, 8)}$$

by writing the self-field $\{F_p^{\mu\nu}\}$ as a sum of the bound velocity fields $\{F_b^{\mu\nu}\}$ and the emitted fields $\{F_r^{\mu\nu}\}$:

$$F_p^{\mu\lambda} = F_b^{\mu\lambda} + F_r^{\mu\lambda}, \qquad \text{(III, 9)}$$

where

$$F_b^{\mu\lambda} = (Z/R^2) (u^\mu v^\lambda - u^\lambda v^\mu) \qquad \text{(III, 10a)}$$

$$F_r^{\mu\lambda} = (Z/R) [\dot{u}^\mu n^\lambda - \dot{u}^\lambda n^\mu - (v \dot{u})(u^\mu v^\lambda - u^\lambda v^\mu)]. \qquad \text{(III, 10b)}$$

The bound field tensor $\{T_b^{\mu\nu}\}$ in (III, 7) then contains the bound field terms and mixed terms, whereas the radiation field tensor $\{T_r^{\mu\nu}\}$ is constituted by the radiation fields $\{F_r^{\mu\nu}\}$ alone. This splitting of the tensors was introduced by Teitelboim[7] for the first time.

Now the integrations in (III, 7) and (III, 8) can easily be performed (e. g. Marx[14]) and one obtains

$$[P_b{}^\mu]_1^2 = \frac{Z^2}{2\,c\,R_0}\,u^\mu{}_{(2)} - \frac{Z^2}{2\,c\,R_0}\,u^\mu{}_{(1)} = \int_1^2 \frac{Z^2}{2\,c\,R_0}\,\dot{u}^\mu{}_{(s)}\,ds\,,$$

(III, 7a)

$$[P_r{}^\mu]_1^2 = -\frac{1}{c}\,\frac{2}{3}\,Z^2 \int_1^2 (\dot{u}\,\dot{u})\,u^\mu{}_{(s)}\,ds \equiv \int_1^2 \frac{dP_r{}^\mu}{ds}\,ds\,.$$

(III, 8a)

The energy-momentum flux of the particle self-fields through the tube surface (t) is now the sum of these two results

$$[P_p{}^\mu]_1^2 = [P_b{}^\mu]_1^2 + [P_r{}^\mu]_1^2\,. \qquad (III, 11)$$

It must be stressed that the results (III, 7a) and (III, 8a) are exact results for an arbitrary value R_0 of the tube radius. Therefore, the change per unit time of the bound energy-momentum $\{P_b{}^\mu\}$ is exactly

$$\frac{dP_b{}^\mu}{ds} = \frac{Z^2}{2\,R_0\,c}\,\dot{u}^\mu{}_{(s)}\,. \qquad (III, 12)$$

So we have actually accomplished our first aim, namely to avoid the emergence of the Schott term in connection with the bound energy-momentum of the particle. The latter can equally well be defined as the light-cone integral

$$P_b{}^\mu{}_{(s)} = \frac{1}{c}\int_{(1)} T_b{}^{\mu\lambda}\,d^3l_\lambda = \frac{Z^2}{2\,c\,R_0}\,u^\mu{}_{(s)}\,, \quad (III, 13)$$

where

$$d^3l^\lambda = n^\lambda\,R^2\,dR\,d\Omega\,(\Theta, \Phi) \qquad (III, 14)$$

is the light-cone surface element. The region of integration is $0 < \Phi < 2\,\pi$; $0 < \Theta < \pi$; $R_0 \leq R < \infty$. Applying Gauß' integral theorem to the four-volume V_4, bounded by the tube surface (t), the distant surface (Σ), and the two light cones $l_{(1)}$ and $l_{(2)}$ (see Fig. 1), and observing Teitelboim's result

$$T_b{}^{\mu\nu}{}_{,\nu} = 0 \quad \text{in } V_4 \qquad (III, 15)$$

yields at once agreement between the two definitions (III, 13) and (III, 7) of the bound four-momentum $\{P_b{}^\mu\}$. A similar consideration applies to the radiated four-momentum $\{P_r{}^\mu\}$.

So far, we have only collected results from various authors. But in computing now the electromagnetic force K^μ

$$[P_{int}^\mu]\,{}_2^1 = -\frac{1}{c}\int_1^2{}_{(t)} T_{int}^{\mu\nu}\,d^3f_\nu \equiv -\frac{1}{c}\int_1^2 K^\mu{}_{(s)}\,ds\,,$$

(III, 16)

exerted on the particle by the external fields $\{F_e{}^{\mu\nu}\}$, we are taking a new route. Since we must not forget that we are dealing with a particle of finite size (R_0 does not tend to zero), it will become necessary to take account of the finite size in the force expression (III, 16). This force shall be calculated now up to powers $(R_0)^1$ inclusively. Nevertheless, we do not want to bring in the variability of the external field $\{F_e{}^{\mu\nu}\}$ over the range of extension of the electron; i. e. we shall replace the values of $\{F_e{}^{\mu\nu}\}$ on the tube surface (t) by the values, which are assumed by $\{F_e{}^{\mu\nu}\}$ on the world line $x^\lambda = z^\lambda{}_{(s)}$. This means that we do not take into account the variation of $\{F_e{}^{\mu\nu}\}$ in a space-time volume of magnitude $(R_0)^4$. As a consequence, we can cast the $F_e{}^{\mu\nu}$-terms in $T_{int}^{\mu\nu}$ of formula (III, 16) in front of the integral and are then left with integrations involving only the self-fields of the electron; for the same reason, we approximate

$$R_0\,\frac{d}{ds}\,(F_e{}^{\mu\lambda}\,u_\mu) \approx R_0\,F_e{}^{\mu\lambda}\,\dot{u}_\mu\,. \qquad (III, 17)$$

Instead of computing (III, 16) directly, we resort to a shorter procedure involving Gauß' integral theorem and the vanishing of the divergence of $T_{int}^{\mu\nu}$ off the world line:

$$T_{int,\,\nu}^{\mu\nu} = 0\,. \qquad (III, 18)$$

Imagine R_0 not being a constant and substitute $R_0 \to R$, where $0 < R < R_0$ in the defining equations of the tube (t) in Sect. II. Holding the two light cones $l_{(1)}$ and $l_{(2)}$ of Fig. 1 fixed, we can consider two tube surfaces (t') and (t''), which have $0 < R' < R'' = R' + \Delta R$ and are otherwise constructed analogously to the surface $R = R_0$; i. e. (t') has $R' = \text{const}$ and (t'') has $R'' = \text{const}$ (see Fig. 2). The four-volume, enclosed by the surfaces $l_{(1)}$, $l_{(2)}$, (t'), and (t''), is designated by ΔV_4. Applying Gauß' integral theorem to this volume ΔV_4, one finds on account of (III, 18)

$$0 = \int_{\Delta V_4} T_{int,\nu}^{\mu\nu}\,d^4x = \oint_{(\sigma)} T_{int}^{\mu\nu}\,d^3\sigma_\nu = \int_{\Delta^3 l_{(2)}} T_{int}^{\mu\nu}\,d^3l_{\nu(2)} \quad (III, 19)$$

$$- \int_{\Delta^3 l_{(1)}} T_{int}^{\mu\nu}\,d^3l_{\nu(1)} - \int_1^2 T_{int}^{\mu\nu}\,d^3f_\nu'' + \int_1^2 T_{int}^{\mu\nu}\,d^3f_\nu'\,.$$

The surface elements $\{d^3f_\lambda'\}$ and $\{d^3f_\lambda''\}$ of the auxiliary tubes (t') and (t'') are constructed quite analogously as (III, 5). Using the definition of the external force $\{K^\mu{}_{(s)}\}$ from (III, 16) in terms of $\{T_{int}^{\mu\nu}\}$, we readily find from (III, 19)

$$K''^\mu{}_{(s)} - K'^\mu{}_{(s)} = \frac{d}{ds}\int_{\Delta^3 l_{(s)}} T_{int}^{\mu\nu}\,d^3l_{\nu(s)}\,, \quad (III, 20)$$
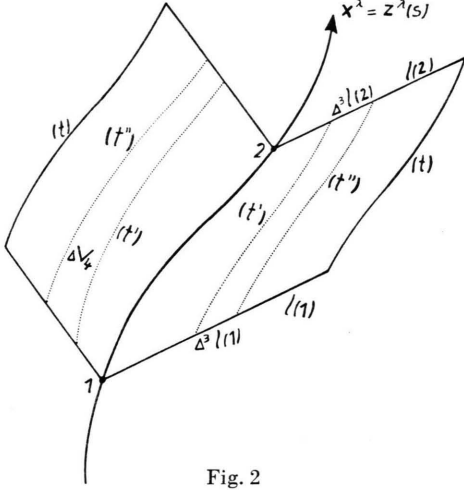
Fig. 2

where $\{K''^\mu\}$ refers to the tube (t″) and $\{K'^\mu\}$ to the tube (t′). Now we let R' of (t′) tend to zero and R'' of (t″) tend to R_0, so that (t′) contracts on to the world line and (t″) coincides with (t). In this case

$$K'^\mu_{(s)} \rightarrow Z F^{\lambda\mu}_{e(s)} u_{\lambda(s)} , \qquad \text{(III, 21)}$$

and therefore

$$K^\mu_{(s)} = Z F^{\lambda\mu}_e u_\lambda + \frac{\mathrm{d}}{\mathrm{d}s} \int_{R'=0}^{R_0} \int_{\Omega(\Theta,\Phi)} \int T^{\mu\nu}_{\mathrm{int}} \, \mathrm{d}^3 l_{\nu(s)} \quad \text{(III, 22)}$$

with $\mathrm{d}^3 l_{\nu(s)}$ taken from (III, 14). So we have obtained a simple prescription of computing that part $\{\Delta K^\mu_{(s)}\}$ of the external force, which surpasses the usual Lorentz term by

$$\Delta K^\mu = \frac{\mathrm{d}}{\mathrm{d}s} \int_{R'=0}^{R_0} \iint_\Omega T^{\mu\nu}_{\mathrm{int}} \, \mathrm{d}^3 l_{\nu(s)} . \qquad \text{(III, 23)}$$

Remembering of our approximation assumptions specified below formula (III, 16), we rewrite (III, 23) approximately as

$$\Delta K^\mu = -\frac{1}{4\pi} \frac{\mathrm{d}}{\mathrm{d}s} \{ F_e{}^\nu{}_\lambda \iiint F_\mathrm{p}{}^{\mu\lambda} \, \mathrm{d}^3 l_\nu \qquad \text{(III, 24)}$$

$$+ F_e{}^{\mu\lambda} \iiint F_\mathrm{p}{}^\nu{}_\lambda \, \mathrm{d}^3 l_\nu - \tfrac{1}{2} F_{e\varrho\sigma} \iiint F_\mathrm{p}{}^{\varrho\sigma} \, \mathrm{d}^3 l^\mu \} ,$$

where the explicit dependence of $\{T^{\mu\nu}_{\mathrm{int}}\}$ from the particle fields and external fields has been inserted. From (III, 24) it is recognized that the computation of the integral

$$\int_{R=0}^{R_0} R^2 \, \mathrm{d}R \iint_\Omega \mathrm{d}\Omega \, F_\mathrm{p}{}^{\mu\lambda} n^\nu \equiv J^{\mu\lambda\nu} \qquad \text{(III, 25)}$$

becomes necessary. But since we want to have the integral $J^{\mu\lambda\nu}$ only up to powers $(R_0)^1$ inclusively,

we can approximate it by

$$J^{\mu\lambda\nu} \approx \int_{R=0}^{R_0} R^2 \, \mathrm{d}R \iint_\Omega \mathrm{d}\Omega \, F_\mathrm{b}{}^{\mu\lambda} n^\nu . \qquad \text{(III, 26)}$$

Inserting here from (III, 10a) yields

$$J^{\mu\lambda\nu} \approx Z R_0 \iint \mathrm{d}\Omega \, (u^\mu v^\lambda v^\nu - u^\lambda v^\mu v^\nu)$$

$$= \frac{4\pi}{3} Z R_0 (u^\lambda g^{\nu\mu} - u^\mu g^{\nu\lambda}) . \qquad \text{(III, 27)}$$

With this result (III, 24) becomes

$$\Delta K^\mu = \frac{\mathrm{d}}{\mathrm{d}s} [Z R_0 F_e{}^{\lambda\mu} u_\lambda + O_{(R_0^2)}] . \quad \text{(III, 28)}$$

The remainder $O(R_0^2)$ is solely due to the radiative part $\{F_\mathrm{r}{}^{\mu\lambda}\}$ in (III, 25), whereas the first term on the right of (III, 28) stems solely from the interaction of the bound self-field $\{F_\mathrm{b}{}^{\mu\lambda}\}$ with the external field $\{F_e{}^{\mu\lambda}\}$. According to the approximation assumption (III, 17) the force on the electron becomes finally

$$K^\mu_{(s)} = Z F_e{}^{\lambda\mu} u_\lambda + Z R_0 F_e{}^{\lambda\mu} \dot{u}_\lambda + O_{(R_0^2)} \quad \text{(III, 29)}$$

and hence the total flux through the tube surface (t) between points 1 and 2 is given by (III, 1) as

$$[P^\mu_{\mathrm{tot}}]^2_1 = [P_\mathrm{b}{}^\mu]^2_1 + [P_\mathrm{r}{}^\mu]^2_1 + [P^\mu_{\mathrm{int}}]^2_1 \qquad \text{(III, 30)}$$

$$= \int_1^2 \frac{Z^2}{2 c R_0} \dot{u}^\mu_{(s)} \, \mathrm{d}s - \frac{1}{c} \int_1^2 \frac{2}{3} Z^2 (\dot{u} \, \dot{u}) u^\mu_{(s)} \, \mathrm{d}s$$

$$- \frac{1}{c} \int_1^2 Z F_{e(s)}^{\lambda\mu} u_{\lambda(s)} \, \mathrm{d}s - \frac{1}{c} \int_1^2 Z R_0 F_{e(s)}^{\lambda\mu} \dot{u}_{\lambda(s)} \, \mathrm{d}s .$$

IV. The Equation of Motion

As we have mentioned briefly in the preceding section, the electromagnetic flux of energy-momentum through the particle's surface (t) can be interpreted as the change of the three sorts of energy-momentum connected with the particle: $[P_\mathrm{b}{}^\mu]^2_1$ is the change of the bound four-momentum between events 1 and 2; $[P_\mathrm{r}{}^\mu]^2_1$ is the corresponding change of the radiated four-momentum; and $[P^\mu_{\mathrm{int}}]^2_1$ is the change of the interaction four-momentum. Usually [5-8], one adds to these the mechanical part of the electron four-momentum

$$P^\mu_{\mathrm{mech}} = m_{\mathrm{mech}} c u^\mu \qquad \text{(IV, 1)}$$

and obtains then the electron's equation of motion by evoking the energy-momentum conservation law for each point of the world line:

$$\frac{\mathrm{d}P^\mu_{\mathrm{mech}}}{\mathrm{d}\tau} + \frac{\mathrm{d}P_\mathrm{b}{}^\mu}{\mathrm{d}\tau} + \frac{\mathrm{d}P_\mathrm{r}{}^\mu}{\mathrm{d}\tau} = - \frac{\mathrm{d}P^\mu_{\mathrm{int}}}{\mathrm{d}\tau} \equiv K^\mu . \quad \text{(IV, 2)}$$

Substituting herein the results (III, 30) and the assumption (IV, 1) one readily arrives at the equation of motion

$$\left(m_{\text{mech}} + \frac{Z^2}{2\,c^2\,R_0}\right) c^2\,\dot{u}^\lambda - \tfrac{2}{3}\,Z^2\,(\dot{u}\,\dot{u})\,u^\lambda \qquad \text{(IV, 3)}$$
$$= Z\,F_{\text{e}}^{\nu\lambda}\,u_\nu + Z\,R_0\,F_{\text{e}}^{\nu\lambda}\,\dot{u}_\nu\,.$$

Now, contract this equation with $\{u_\lambda\}$ to find

$$-\tfrac{2}{3}\,Z^2\,(\dot{u}\,\dot{u}) = Z\,R_0\,(F_{\text{e}}^{\nu\lambda}\,\dot{u}_\nu\,u_\lambda) \qquad \text{(IV, 4)}$$

and resubstitute this into (IV, 3) to obtain finally

$$m_{\text{obs}}\,c^2\,\dot{u}^\lambda + Z\,R_0\,(F_{\text{e}}^{\nu\mu}\,\dot{u}_\nu\,u_\mu)\,u^\lambda \qquad \text{(IV, 5)}$$
$$= Z\,F_{\text{e}}^{\nu\lambda}\,u_\nu + Z\,R_0\,F_{\text{e}}^{\nu\lambda}\,\dot{u}_\nu\,.$$

In order to establish a relation between the observable mass m_{obs}

$$m_{\text{obs}} = m_{\text{mech}} + Z^2/2\,R_0\,c^2 \qquad \text{(IV, 6)}$$

and the size parameter R_0, we consider the one-dimensional motion parallel to x^3-axis, which can be characterized by

$$\{u^\lambda_{(s)}\} = \{\text{Cosh}\,w_{(s)}\,;\,0,0,\,\text{Sinh}\,w_{(s)}\}\,, \qquad \text{(IV, 7)}$$

$$\{Z\,F_{\text{e}}^{\nu\lambda}\,u_\nu\} = Z\,E\,\{\text{Sinh}\,w\,;\,0,0,\,\text{Cosh}\,w\}\,. \qquad \text{(IV, 8)}$$

One can easily verify that this type of motion has the proberty

$$(F_{\text{e}}^{\nu\mu}\,\dot{u}_\nu\,u_\mu)\,u^\lambda = F_{\text{e}}^{\nu\lambda}\,\dot{u}_\nu\,, \qquad \text{(IV, 9)}$$

and therefore the trajectory in this case satisfies the neutral particle equation [1, 23]

$$m_{\text{obs}}\,c^2\,\dot{u}^\lambda = Z\,F_{\text{e}}^{\nu\lambda}\,u_\nu\,. \qquad \text{(IV, 10)}$$

On the other hand, Eq. (IV, 4) must be valid for every type of motion. Hence, contraction of (IV, 10) with $\{\dot{u}_\lambda\}$ and comparison of the result with (IV, 4) requires

$$m_{\text{obs}}\,c^2\,R_0 = \tfrac{2}{3}\,Z^2\,. \qquad \text{(IV, 11)}$$

Thus we see that R_0 is identical with the non-locality parameter Δs of References [1, 2]. Using (IV, 11), the final form of (IV, 5) is

$$m_{\text{obs}}\,c^2\,\dot{u}^\lambda + \frac{2}{3}\,\frac{Z^3}{m_{\text{obs}}\,c^2}\,(F_{\text{e}}^{\nu\mu}\,\dot{u}_\nu\,u_\mu)\,u^\lambda \qquad \text{(IV, 12)}$$
$$= Z\,F_{\text{e}}^{\nu\lambda}\,u_\nu + \frac{2}{3}\,\frac{Z^3}{m_{\text{obs}}\,c^2}\,F_{\text{e}}^{\nu\lambda}\,\dot{u}_\nu\,,$$

which is exactly the Mo-Papas equation [4].

It is impressive to list all the disadvantages of the other theories encountered in literature, which have been avoided in the present derivation of the equation of motion (IV, 12) for the radiating electron:

1. No use of advanced fields was necessary.

2. No asymptotic conditions had to be used in the derivation of the equation of motion.

3. No infinite terms arise in the connection with the mass of the particle [cf. Eqs. (IV, 6) and (IV, 11)].

4. No unphysical effects (runaways, pre-acceleration) are present in the resulting equation of motion; and therefore no asymptotic conditions have to be imposed on the solutions artificially.

A final remark has to be made on the force term

$$K^\lambda_{(s)} = Z\,\widehat{F}_{(s)}^{\mu\lambda}\,u_{\mu(s)} \equiv Z\,F_{(s-\Delta s)}^{\mu\lambda}\,u_{\mu(s)} \qquad \text{(IV, 13)}$$

in the exact equation of motion (I, 1). Since the left-hand side of (I, 1) is a vector orthogonal to $\{u^\lambda_{(s)}\}$, the force $\{K^\lambda_{(s)}\}$ must satisfy

$$K^\lambda\,u_\lambda = 0\,. \qquad \text{(IV, 14)}$$

So we could have equally well put

$$K^\lambda_{(s)} \to Z\,F_{(s)}^{\mu\lambda}\,u_{\mu(s)} \qquad \text{(IV, 15)}$$

instead of the choice (IV, 13), which we have preferred in the first paper [2] on this subject. The original motivation for preferring (IV, 13) to (IV, 15) was that whenever the external field $F_{\text{e}(z_{(s)})}^{\mu\lambda}$ vanishes in a special event $\{\check{z}^\lambda\}$ on the world line, then the world line has $\dot{u}^\lambda = 0$ in this special event $\{\check{z}^\lambda\}$. Therefore, the world line cannot be curved before the external force has non-zero values on it. In this sense, one might say that pre-acceleration is avoided in our model.

There exists however a second reason, for which (IV, 13) has to be preferred to (IV, 15). This reason consists in the requirement of a consistent lowest order approximation with respect to the radiation reaction phenomenon. As stated in Sect. I, radiation reaction and non-locality of the equation of motion are intimately connected, and therefore "lowest order approximation of radiation reaction" means an expansion of the equation of motion with respect to the non-locality parameter Δs. Such an expansion was performed in the previous paper [1] and one has found the Mo-Papas Eq. (IV, 12) instead of the Lorentz-Dirac Equation (I, 2). But for the emergence of the Mo-Papas equation it is essential that the force term in (I, 1) is given by (IV, 13) and not by (IV, 15). Would it be given by (IV, 15), then there would arise the additional term $Z\,\Delta s\,\dot{F}^{\mu\lambda}\,u_\mu$ in the Mo-Papas equation (note that expansions have to be performed about $\{\check{z}^\lambda_{(s)}\} \equiv \{z^\lambda_{(s-\Delta s)}\}$), and this new term would destroy the consistency of

the equation. So we see that (IV, 13) is indeed stringent.

It is true, we have discarded such a term also in the foregoing derivation of the Mo-Papas equation by Dirac's method [observe (III, 17)] but in that context this was allowed, because we had neglected the variability of $\{F_e{}^{\mu\nu}\}$ in a four-volume of magnitude $(R_0)^4$. Moreover, we could have used the direct method for computing $\{\Delta K^\lambda\}$, i. e. not ap-

plying Gauß' integral theorem [see the argumentation below (III, 17)], and then we would have found the result (III, 29) without the approximation assumption (III, 17)!

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